

FACULTY OF ELECTRONICS

**SUBJECT CARD****Name in Polish** Metody matematyczne automatyki i robotyki**Name in English** Mathematical methods of automation and robotics**Main field of study (if applicable):** Control Engineering and Robotics**Level and form of studies:** 2nd level, full-time**Kind of subject:** obligatory**Subject code** AREA17002**Group of courses** YES

	Lecture	Classes	Laboratory	Project	Seminar
Number of hours of organized classes in University (ZZU)	30	30			
Number of hours of total student workload (CNPS)	80	100			
Form of crediting	examination	crediting with grade			
For group of courses mark (X) final course	X				
Number of ECTS points	6				
including number of ECTS points for practical (P) classes		3			
including number of ECTS points for direct teacher-student contact (BK) classes	2	2			

\*delete as applicable

**PREREQUISITES RELATING TO KNOWLEDGE, SKILLS AND OTHER COMPETENCES**

1. None

**SUBJECT OBJECTIVES**

- C1. Gain knowledge of mathematical methods of modern automation and robotics
- C2. Peruse the paradigm of transformation and equivalence
- C3. Learn of properties and equivalence of functions
- C4. Learn of properties and equivalence of dynamical systems
- C5. Learn of properties and feedback equivalence of control systems
- C6. Learn of synthesis of control algorithms for linearizable, decouplable, and differentially flat systems
- C7. Learn of using normal forms in the synthesis of control algorithms

### SUBJECT EDUCATIONAL EFFECTS

relating to knowledge:

PEK\_W01- knows pillars of nonlinear analysis: inverse function theorem, theorem of existence and uniqueness of trajectory of dynamic system, Frobenius theorem, and contraction function theorem

PEK\_W02 - knows the concept of equivalence of functions and their normal forms

PEK\_W03 - knows the concept and properties of dynamic system

PEK\_W04 - knows the definition of equivalence of dynamic systems and basic theorems on the equivalence

PEK\_W05 - knows the concept and properties of control affine system

PEK\_W06 - knows the concept of feedback equivalence of control systems

PEK\_W07 - knows methods of synthesis of control algorithms based upon linearization or decoupling by static feedback

PEK\_W08 - knows methods of synthesis of control algorithms based upon dynamic feedback linearization

PEK\_W09 - knows the concept of differentially flat system and its applicability in synthesis of control algorithms

PEK\_W10 - knows applicability of normal forms in synthesis of control algorithms

relating to skills:

PEK\_U01- can use the pillar theorems of nonlinear analysis

PEK\_U02 - can use the implicit function theorem in the context of robot manipulator kinematics

PEK\_U03 - can make use of theorems of immersions, submersions, and Morse functions, understands the concept of kinematic singularities of robotic manipulators

PEK\_U04 - can analyze properties of dynamic systems

PEK\_U05- can make use of theorems on equivalence of dynamic systems, understands their connection to Lyapunov theorems

PEK\_U06 - can make use of Lie brackets as a tool of analysis of nonlinear control systems

PEK\_U07 - can use theorems on feedback linearization and decoupling of control systems, understands the role of these methods in synthesis of control algorithms

PEK\_U08- can make use of differential flatness in control of mobile robots

PEK\_U09 - can make use of normal forms in synthesis of robot control algorithms

PEK\_U10 – can apply mathematical methods in synthesis of control algorithms of diverse systems of automation and robotics

relating to social competences:

PEK\_K01- understands significance of information retrieval and critical analysis

PEK\_K02 - can debate, rationally explain, and justify his/her own standpoint relying on the subject knowledge

PEK\_K03 – understands significance of mathematical methods in automation in robotics

### PROGRAMME CONTENT

Form of classes - lecture		Number of hours
Lec 1	Smooth functions, inverse function theorem, diffeomorphism	2
Lec 2	Newton algorithm	2

Lec 3	Implicit function theorem	2
Lec 4	Equivalence of functions, normal forms	2
Lec 5	Dynamic system, existence and uniqueness theorem, contraction function theorem	2
Lec 6	Stability of dynamic systems	2
Lec 7	Equivalence of dynamic systems, theorems on linearization	2
Lec 8	Vector fields, Lie brackets, distributions, Frobenius theorem	2
Lec 9	Control affine systems, controllability	2
Lec 10	Feedback equivalence	2
Lec 11	Linearization by static feedback	2
Lec 12	Input/output decoupling, zero dynamics	2
Lec 13	Linearization by dynamic feedback	2
Lec 14	Differential flatness	2
Lec 15	Nonlinear normal forms	2
	Total hours	30

<b>Form of classes - class</b>		<b>Number of hours</b>
Cl 1	Feedback equivalence of linear control systems: Brunovsky canonical form	2
Cl 2	Matrix norms	2
Cl 3	Inverse and implicit function theorems	2
Cl 4	Immersion, submersions, Morse functions	2
Cl 5	Equivalence of dynamic systems	2
Cl 6	Stability analysis of dynamic systems	2
Cl 7	Gradient and Hamiltonian systems	2
Cl 8	Control systems: definition and properties of Lie brackets	2
Cl 9	Feedback equivalence and linearization	2
Cl 10-11	Analysis of linearization conditions, equations of equivalence	4
Cl 12	Differential degree, input/output decoupling, zero dynamics	2
Cl 13	Analysis of differential flatness	2
Cl 14	Nonlinear normal forms	2
Cl 15	Test	2
	Total hours	30

<b>TEACHING TOOLS USED</b>
N1. Traditional lecture N2. Classes N3. Consultations N4. Independent work – solving example problems N5. Independent work – literature study

## EVALUATION OF SUBJECT EDUCATIONAL EFFECTS ACHIEVEMENT

Evaluation (F – forming (during semester), P – concluding (at semester end))	Educational effect number	Way of evaluating educational effect achievement
F1	PEK_W01 ÷ PEK_W10;	examination
F2	PEK_W01 ÷ PEK_W10; PEK_U01 ÷ PEK_U10;	active participation in classes, test
C=0.4*F1+0.6*F2		

Notice: a mark at least 3.0 (passed) within F2 is prerequisite of admission to the exam F1.

### PRIMARY AND SECONDARY LITERATURE

#### **PRIMARY LITERATURE:**

- [1] M. Golubitsky, V. Guillemin: „Stable Mappings and Their Singularities”, Springer-Verlag, New York, 1974.
- [2] R. Abraham, J. E. Marsden, T. Ratiu: „Manifolds, Tensor Analysis, and Applications”, Springer-Verlag, New York, 1988.
- [3] V. I. Arnold: „Geometrical Methods in the Theory of Ordinary Differential Equations”, Springer-Verlag, New York, 1983.
- [4] S. S. Sastry: „Nonlinear Systems”, Springer-Verlag, New York, 1999.
- [5] A. M. Bloch: „Nonholonomic Mechanics and Control”, Springer-Verlag, New York, 2003.
- [6] H. Nijmeijer, A. J. van der Schaft: „Nonlinear Dynamical Control Systems”, Springer-Verlag, New York, 1990.
- [7] H. Sira-Ramirez, S. K. Agrawal: „Differentially Flat Systems”, Marcel Dekker, New York, 2004.
- [8] **K. Tchoń, R. Muszyński: Lecture Notes available on the Internet**

#### **SECONDARY LITERATURE:**

- [1] Ph. Hartman: „Ordinary Differential Equations”, J. Wiley, New York, 1964.
- [2] H. K. Khalil: „Nonlinear Systems”, Prentice-Hall, New Jersey, 2000.
- [3] R. Murray, Z. Li, S. S. Sastry: „A Mathematical Introduction to Robotic Manipulation”, CRC Press, Boca Raton, 1994.
- [4] A. Isidori: „Nonlinear Control Systems”, Springer-Verlag, New York, 1995.
- [5] V. Jurdjevic: „Geometric Control Theory”, Cambridge Univ.Press, Cambridge, 1997.
- [6] J. Levine: „Analysis and Control of Nonlinear Systems: A Flatness-based Approach”, Springer-Verlag, Berlin, 2009.

#### **SUBJECT SUPERVISOR (NAME AND SURNAME, E-MAIL ADDRESS)**

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**MATRIX OF CORRELATION BETWEEN EDUCATIONAL EFFECTS FOR SUBJECT  
Mathematical methods of automation and robotics  
AND EDUCATIONAL EFFECTS FOR MAIN FIELD OF STUDY  
Control Engineering and Robotics**

Subject educational effect	Correlation between subject educational effect and educational effects defined for main field of study and specialization (if applicable)**	Subject objectives***	Programme content***	Teaching tool number***
<b>PEK_W01 (knowledge)</b>	K2AIR_W07,K2AIR_W09	C1, C7	Lec1-Lec5, Lec9	1,3,4,5
<b>PEK_W02</b>	K2AIR_W07	C2, C3	Lec4	1,3,4,5
<b>PEK_W03</b>	K2AIR_W07,	C4	Lec5	1,3,4,5
<b>PEK_W04</b>	K2AIR_W07	C2, C4	Lec6-Lec7	1,3,4,5
<b>PEK_W05</b>	K2AIR_W07	C5	Lec8-Lec9	1,3,4,5
<b>PEK_W06</b>	K2AIR_W07,	C2, C5	Lec10	1,3,4,5
<b>PEK_W07</b>	K2AIR_W07,K2AIR_W09	C6	Lec11-Lec12	1,3,4,5
<b>PEK_W08</b>	K2AIR_W07,K2AIR_W09	C6	Lec13	1,3,4,5
<b>PEK_W09</b>	K2AIR_W07,K2AIR_W09	C6	Lec14	1,3,4,5
<b>PEK_W10</b>	K2AIR_W07,K2AIR_W09	C1, C5-C7	Lec10-Lec15	1,3,4,5
<b>PEK_U01 (skills)</b>	K2AIR_U08,K2AIR_U09	C1	C13-C14, C19	2,3,4
<b>PEK_U02</b>	K2AIR_U08	C1, C3	C13	2,3,4
<b>PEK_U03</b>	K2AIR_U08	C2, C3	C14	2,3,4
<b>PEK_U04</b>	K2AIR_U08	C4	C15-C17	2,3,4
<b>PEK_U05</b>	K2AIR_U08	C2, C4	C15	2,3,4
<b>PEK_U06</b>	K2AIR_U08	C2, C5	C18	2,3,4
<b>PEK_U07</b>	K2AIR_U08	C6	C19-C112	2,3,4
<b>PEK_U08</b>	K2AIR_U08	C6	C113	2,3,4
<b>PEK_U09</b>	K2AIR_U08	C2, C7	C114	2,3,4
<b>PEK_U10</b>	K2AIR_U08,K2AIR_U09	C2, C6, C7	C19-C115	2,3,4
<b>PEK_K01- PEK_K03(competences)</b>	K2AIR_K01,K2AIR_K03	C6, C7	Lec1-Lec15, C11-C115	1,2,3,5

\*\* - enter symbols for main-field-of-study/specialization educational effects

\*\*\* - from table above